Universe Scientific Publishing

Water and Hydroelectric Power Sharing

Liangzhou Tang, Peiyi Yu, Yongsheng Lei

Chengdu University of Technology, Chengdu 610000, China.

Abstract: With climate change, the amount of water available to dams and reservoirs in many regions is decreasing. In order to help the US government solve the problem of water shortage in the west, we have formulated a reasonable water allocation plan according to the current water supply conditions. Based on the water levels of Lake Mead and Lake Powell and the water supply and demand relationship in five states, a supply and demand condition model was established to obtain the hydropower demand plan of the five states. The model calculated that it would take 0.356 days without additional water supply. In order to solve the interest competition between general water use and electricity production water, a linear programming model and a satisfaction model are established.

Keywords: Colorado River Basin; Water Allocation; Linear Programming Model

1. Introduction

The Colorado River is the lifeblood of the American Southwest, providing water and hydroelectric power to 40 million people in seven states in the region. Extreme climates, such as extreme heat, extreme drought, extreme precipitation, extreme low temperatures, have caused the water levels at Glen Canyon Dam (Lake Powell) and Hoover Dam (Lake Mead) to continue to drop.

In recent years, the water storage capacity of the two lakes has reached an all-time low. In 2021, the water storage capacity of Lake Powell is only 30% of the design capacity, and the water storage capacity of Lake Mead is only 35% of the design capacity, while the Colorado River is in the middle of the two lakes.[1] In order to solve the problem of supply (water availability) and demand (power demand), it is very important to develop a rational and efficient water distribution plan.

2. Assumptions

To simplify the problem, we make the following basic assumptions, each of which is properly justified. The shape of the water enclosed between the two water surfaces of the contour line is a platform. Water used for power generation in a short period of time cannot be quickly diverted to other uses. In a short period of time, the water and electricity consumption of the five continents is fixed. Our solution does not utilize or rely on any existing historical agreements or current political power of these state organizations or individuals, but represents the best mathematical solution for our team to distribute water in the region.

3. Model 1: Supply and Demand Condition Model

First, based on the analysis of the problem and the relevant data reviewed, we can derive the average daily water demand $a_i(i = 1, 2,3,4,5)$ for Arizona, California, Wyoming, New Mexico and Colorado and the daily electricity consumption $b_i(i = 1, 2,3,4,5)$ of these five states. By assuming that a represents the total daily water usage in the five states and b represents the total daily electricity usage in the five states, we have:

Secondly, for the Glen Dam, we can divide the water resources of this water into 4 parts: the water consumption of the five states, the amount of water used for electricity generation, the amount of water flowing into the Hoover Dam and the amount of water stored in the reservoir, and we have: For the Hoover Dam, we can divide the water of this water into 3 parts: the water used in the five states, the amount of water used for electricity generation, and the water stored in the reservoir.

Therefore, we can get the total water consumption of the five states from the two dams as: $m_1 + n_1$. The total water consumption of the five states cannot be greater than $m_1 + n_1$, so:

$$a = \sum_{i=1}^{5} a_i \le m_1 + n_1 \tag{1}$$

Assuming that the water level of Lake Mead is M and the water level of Lake Powell is P, we have:

$$W_{LM} = \frac{1}{3}\left(A_1 + \sqrt{A_1 A_2} + A_2\right)M + m_3 \tag{2}$$

$$W_{LP} = \frac{1}{3}\left(A_3 + \sqrt{A_3 A_4} + A_4\right) \tag{3}$$

$$W_{LM} = \sum_{i=1}^{3} n_i \tag{4}$$

$$W_{LP} = \sum_{i=1}^{4} m_i \tag{5}$$

From the conservation of energy theorem, we can obtain:

$$W_e = \alpha[m_2 \rho(P - h_1)g + n_2 \rho(M - h_2)g] \tag{6}$$

For the remaining water volume of Lake Powell and Lake Mead, m_4, n_3,which must meet the safe water level specified by them.[2]

$$u_1 \le \frac{m_4}{s_1} \le u_2 \tag{7}$$

$$u_3 \le \frac{n_3}{s_2} \le u_4 \tag{8}$$

By assuming the daily discharge of Lake Mead and Lake Powell, we have:

$$t = \begin{cases} t_1 = \dfrac{m_1}{V_{L_P}}, t_1 \ge t_2 \\ t_2 = \dfrac{n_1}{V_{LM}}, t_1 < t_2 \end{cases} \tag{9}$$

We have a classification discussion of t, and when $t \le 1$, it means that the discharge of two dams can meet the water needs of five states, and when $t > 1$, it means that the discharge of two dams cannot meet the water needs of five states. From this, it can be seen that the actual displacement is $V_{L_P} + V_{LM}$, and the actual demand is $m_1 + n_1$, so we can get the additional water consumption that needs to be replenished on the day as follows:

$$Q_w{}' = m_1 + n_1 - \left(V_{L_P} + V_{LM}\right) \tag{10}$$

So after time t, we can get the amount of water we need to replenish for:

$$Q_w = [m_1 + n_1 - \left(V_{L_P} + V_{LM}\right)]t \tag{11}$$

In summary, we have:

$$Q_w = \begin{cases} 0, t \le 1 \\ [m_1 + n_1 - \left(V_{L_P} + V_{LM}\right)]t, t > 1 \end{cases} \tag{12}$$

According to the above analysis, the greater the power generation in each state, the higher the efficiency. Therefore, the more water resources allocated, the better. If no additional water is provided, the two dams would need to meet the five states' own water reserves in addition to the five states' general water use. Usually, water reserves are mainly met by rainfall. By optimizing the model, the state's satisfaction with hydropower resources is established with the lowest standard, and our highest satisfaction is

$$y_{\max}=\frac{\alpha[m_2\rho(P-h_1)g+n_2\rho(M-h_2)g]-\sum_{i=1}^{5}b_i}{\sum_{i=1}^{5}b_i}+\frac{m_1+n_2-\left(\sum_{i=1}^{5}a_i+\sum_{i=1}^{5}Q_i\right)}{\sum_{i=1}^{5}a_i+\sum_{i=1}^{5}Q_i} \tag{13}$$

$$\alpha[m_2\rho(P-h_1)g+n_2\rho(M-h_2)g]>\sum_{i=1}^{5}b_i$$

$$\frac{1}{3}(A_1+\sqrt{A_1A_2}+A_2)M+m_3=n_1+n_2+n_3$$

$$\frac{1}{3}(A_3+\sqrt{A_3A_4}+A_4)=m_1+m_2+m_3+m_4$$

$$\sum_{i=1}^{5}a_i+\sum_{i=1}^{5}Q_i\le m_1+n_2 \tag{14}$$

$$u_1\le\frac{m_4}{s_1}\le u_2$$

$$u_3\le\frac{n_3}{s_2}\le u_4$$

$$m_i\ge 0;n_i\ge 0$$

By referring to the relevant data and bringing it into the code of the supply and demand relationship model we constructed, we can get the distribution demand of water and electricity in five states.[3][4]

Take the discharge of Lake Mead and Lake Powell into Equation 13 and Equation 14, and then put the results into Equation 15 for comparison to determine the value of t. Because $t<1$, it would take 0.365 days to meet the water needs of the five states without an additional water supply. Comparing the ratios of industrial, agricultural, and residential water use to daily water discharge in the five states of Lake Powell and Lake Mead, a maximum time t=0.36, t<1, indicates that the dam discharge can meet the water needs of the five states, Its monthly water demand is equal to 0.36 days of water discharge from Lake Mead and Lake Powell, while allowing for more water storage and water use for power generation.

4. Model 2: Benefit Linear Programming Model and Satisfaction Model

Based on the above model, the general (agricultural, industrial, residential) water and electricity production is addressed if the competitive interests in water availability are taken into account. So, the total profit generated by Lake Powell and Lake Mead for general (agricultural, industrial, residential) water use, we have:

$$P_1=k_1(m_1+n_1) \tag{1}$$

The total profit generated by the actual power generation of Lake Powell and Lake Mead, we have:

$$p_2=k_2\alpha[m_2\rho(P-h_1)g+n_2\rho(M-h_2)g] \tag{2}$$

In order to maximize profits while meeting the water needs of five states, we build a target planning model based on Model 1, we have:

$$y_{\max}=\frac{\alpha[m_2\rho(P-h_1)g+n_2\rho(M-h_2)g]-\sum_{i=1}^{5}b_i}{\sum_{i=1}^{5}b_i}+\frac{m_1+n_2-\sum_{i=1}^{5}a_i}{\sum_{i=1}^{5}a_i} \tag{3}$$

$$P_{\max}=k_1(m_1+n_1)+k_2\alpha[m_2\rho(P-h_1)g+n_2\rho(M-h_2)g] \tag{4}$$

By looking up the information about electricity profit and bringing its data into formula 26, the maximum value of profit can be obtained. According to formula 25 and formula 27, the maximum satisfaction can be obtained. And according to it, the optimal water resource allocation plan is formulated. According to the relevant data of general (agricultural, industrial, residential) water use and electricity production water use in five states, the different proportions of the five states are used as the standards of competing interests. Through the benefit linear programming model and the satisfaction model, we can illustrate that the water resources in the Glen Dam are mainly used for the general water needs of the five states, and the water resources in the Hoover Dam are mainly used for storage in the Lake Mead Reservoir.

5. Model 3: Additional Water Use Linear Programming Model and Satisfaction Model

If the current water resources for power generation are insufficient, based on Model 1, we have:

$$Q_{as} = \alpha[m_2\rho(P - h_1)g + n_2\rho(M - h_2)g] \quad (1)$$

$$Q_{ans} = \sum_{i=1}^{5} b_i \quad (2)$$

$$Q_r = Q_{ans} - Q_{as} = \sum_{i=1}^{5} b_i - \alpha[m_2\rho(P - h_1)g + n_2\rho(M - h_2)g] \quad (3)$$

Under the condition that other water uses in the five states can be determined to make Q_r the smallest water allocation scheme, we have:

$$y_{\max} = \frac{\alpha[m_2\rho(P - h_1)g + n_2\rho(M - h_2)g] - \sum_{i=1}^{5} b_i}{\sum_{i=1}^{5} b_i} + \frac{m_1 + n_2 - \sum_{i=1}^{5} a_i}{\sum_{i=1}^{5} a_i} \quad (4)$$

$$Q_{r\,min} = \sum_{i=1}^{5} b_i - \alpha[m_2\rho(P - h_1)g + n_2\rho(M - h_2)g] \quad (5)$$

According to the above model and known data, using Equation 31 and Equation 32, the optimal water resource allocation scheme that satisfies the basic water consumption of the five states can be obtained when the additional water consumption is the minimum value.List the functions that need additional supplementary water according to linear programming, and then write the constraints according to the law of conservation of mass and energy conservation, etc., and then write the satisfaction function, and use matlab to solve the minimum value of additional supplementary water. The corresponding parameter value. When m_1, m_2, m_3, m_4, n_1, n_2, n_3 are the above values, the additional water demand function takes the minimum value. At this time, the need for additional supplementary water is the least, and the purpose of water saving is achieved.

Conclusion

In our article, we design a water resource allocation scheme and build a supply and demand model, a linear programming model, and a satisfaction model to formulate the scheme. The objective function was determined by combining local monthly rainfall, water level data from dams, reservoir storage, and general water use in five states. The constraint conditions are determined according to the law of conservation of mass and energy, and the water supply demand relationship without additional water supply is obtained. In addition, the optimization model derives the optimal allocation of water resources in the five states through the competing interests of water resources and electricity production in the five states. On this basis, we take into account the impact of real-world conditions such as extreme weather, and the model redistributes the overall hydropower demand in the five states under water scarcity scenarios. Finally, combined with the influence of time and other variables, the model was adjusted to the allocation of water resources. Since some assumptions in our model may cause the model to be idealized to a certain extent, there is still a certain gap with reality. In the future, we still have the opportunity to continue optimizing our model.

References

[1] Liu, D., After a 22-year drought: The American West is "drying up." Provided from: https://www.thepaper.cn/ .

[2] Baidu Encyclopedia. Lake Mead, Lake Powell. Provided from: https://baike.baidu.com/ .

[3] U.S. National Electricity Profile. State Electricity Profiles. Provided from: https://www.eia.gov/electricity/state/

[4] NOAA. Monthly rainfall around. Provided from: https://www.noaa.gov/

[5] GB/T 33590.2-2017 Technical Specifications for Smart Grid Dispatching and Control Systems Part 2: Terminology

[6] Mathematical Modeling Methods and Analysis (Original Book 4th Edition) / (US) Meerschaert, M. M. Translated by Liu Laifu, Huang Haiyang, Yang Chun.—Beijing: Machinery Industry Press, 2014.12:1-9.