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Matrix Equations Under Semi-Tensor ProductAx=B; XC=DLeast Squares Solution

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Abstract: In this paper, we study the matrix equations under the semi-tensor product.Ax=B; XC=DThe least squares solution under Different ConditionsX$^{The\ following}$2.R$^{P£q}$,Where MatrixA 2R$^{M\ £\ n}$B 2R$^{H\ £\ K}$C 2R$^{A\ £\ B}$D 2R$^{L\ £\ d}$Given.According to the definition of semi-tensor product, it is transformed into matrix equations under ordinary product.,Combined with the Matrix Singular Value Decomposition and matrix differential, the analytic expression of the least squares solution of the equations under different circumstances is given.,And it is verified by numerical examples..

Keywords: Semi-Tensor Product;Matrix Equations;Least Squares Solution;Singular Value Decomposition

1. Introduction

Notation used in this articleR$^{M\ £\ n}$Represent all real DomainsM£NSet of order Matrices;I$_K$ForKOrder unit matrix.

M 2R$^{M\ £\ n}$,M^{T}AndM^{Y}Transpose andMoore-PenroseGeneralized Inverse.Pair MatrixM; N 2R$^{M\ £\ n}$,Inner Product is definedHM; Ni= Trace (M^{T}N),The resulting matrix norm isFrobeniusNorm,MarkK. K..LCMFM; ngAndGCDFM; ngPositive IntegerM; nThe smallest common multiple and the largest common divisor.Okay.M= [A$_{IJ}$],N= [B$_{IJ}$]2.R$^{M\ £\ n}$,M-NRepresentation MatrixMAndNOfKroneckerJi[1]

M 'nRepresentation MatrixMWithNOfHadamardJi[1]

Okay.M= [A$_{IJ}$]2.R$^{M\ £\ n}$,Column straighten operator of MatrixV$_C$(Wang Yi)And line straighten OperatorV$_R$(Wang Yi)Expressed separately

V$_C$(M) = [A$_{11}$A$_{21}$.A.$_{M1}$.A.$_{1.N}$A$_{2.N}$A.$_{Mn}$]^T;

V$_R$(M) = [A$_{11}$A$_{12}$A.$_{1.N}$A.$_{M1}$.A$_{M2}$.A.$_{Mn}$]:

Definition1.[2]Given MatrixA2.R$^{M\ £\ n}$; B2.R$^{H\ £\ K}$,JiT= LCMFN; HgForN; HMinimum common multiple,MatrixAAndBThe semi-tensor product

ANB= (A-I$_{T = N}$)(B-I$_{T = H}$)2.R$^{Mt = n\ £\ KT = H}$:

The semi-tensor product was originally proposed by Professor CHENG daizhan to solve the matrix representation problem of multiple linear functions.[3],Then it is not only applied to the array of high dimensional data, but also to the algebraic Control of Power System Nonlinear Robust Stability.[4],And for the Boolean Network[5],Cryptography[6],Graph Coloring[7],Fuzzy Control[8]Problem Research provides a new research tool in the field.In some cases, the solution of these problems can be attributed to the Solution of Linear Equations or matrix equations under the semi-tensor product..For example, in the non-cooperative network problem[9],FeatureMIndividual player,JiM=F1.;2.;My wife and I; Mg,PlayerJThe policy set isN=F1.; N$_J$G; j= 1;2.M:Assume playerJThe mixed strategy is$_X$NJ

$A^I{}_J = 1; J = 1;2.(M;$

$I = 1$

What you wantNashEquilibrium Point(A$^{The\ following}{}_{1.}$A$^{The\ following}{}_{2.}$;My wife and IA$^{The\ following}{}_{M}$)It is equivalent to solving the matrix equation under the following semi-Tensor Product

N$^{M}{}_{J=1}$A$^{The\ following}{}_{J}$= 0;

Among them©Known.For such issues,Yao Juan[10]It is reduced to a matrix equation under the semi-tensor product.ANX=BSolving Problems,The necessary and sufficient condition for the solution of the equation under the semi-tensor product and the specific analytical expression are studied in detail.,In his doctoral dissertation, the matrix equation under the semi-tensor product is discussed.ANX=BLeast Squares solution.In practical application,Rumble Network,Fuzzy Control and network non-cooperation,We often encounter more complicated problems.,Using the semi-Tensor Product Tool,This problem can be attributed to the following matrix equations under the semi-Tensor Product

Equation(1.1)The coefficient matrix in this paper comes from the measured data.,Due to the measurement error,The data is always inaccurate..Also due to rounding error,Equation(1.1)Not necessarily satisfied similar to the text[10]The more complex compatibility condition given,Therefore, it is necessary to discuss the least square problem of matrix equations under the following semi-tensor product..

Problem1.Given MatrixA2.R$^{M £ n}$,B2.R$^{H £ K}$,C2.R$^{A £ B}$AndD2.R$^{L £ d}$,PleaseX$^{The\ following}$2.R$^{P£q}$Meet

KaNX$^{The\ following}$¡BK$^{2\cdot}$Kx$^{The\ following}$NC ¡dk$^{2\cdot}$= MinKaNX ¡BK$^{2\cdot}$KxNC ¡dk$^{2\cdot}$:(1.2)

2.R$^{P£q}$

For Matrix Equation(Group)Of solving and Its the most square by Solution ProblemBecause its in biologicalEngineering MechanicalParameters RecognitionVibration Theory of Inverse Problem and linear planning and widely of ApplicationSo has been widely Research.Many scholars use matrix blockGeneralized

InverseMatrix decomposition and many kinds of skills methods will equation or equations drop dimensionAnd then given solution there of sufficient to conditions and its specific analysis expressionOr further discussion and the most square by solution and very norm number of the most square by Solution.Such as the general matrix equationsMitra[11]Based on generalized inverse given the very small rank the most square by Solution;Lee[12{14]Respectively explore the equation(1.3)Of self-anti-SolutionGeneralized self-anti-SolutionMirror symmetric most square by SolutionIn ·-Earl mitt the most square by Solution;Jo[15]The useKroneckerProduct will be its into corresponding equationsReuse generalized inverse given solution straightening out of the form;Yuan[16]Use matrix of spectral decomposition are by solution and symmetric most square by solution of display expression.Ordinary matrix product under matrix equations(1.3)Of solving and Its the most square by solution has been fully studyBut semi-tensor product under matrix equations(1.1)There is no research resultsSo Will semi-tensor product concept Promotion to matrix equations of research work is also meaningful.Similar to paper[10]Processing semi-tensor under Matrix EquationAX=BOf research ideasFirst Will semi-tensor product under the matrix equations transformation for ordinary product under of matrix equationsAnd then combined with matrix block, Matrix Generalized Inverse and its matrix decomposition technique to give the most square by solution of specific analysis expression.Same first consider moment

-Vector EquationThe(1.1)-InXFor vector of situationStudy Its the most square by solution of specific analysis expression;And then explore the general form matrix-Matrix EquationThe(1.1)-InXFor matrix of situationStudy Its the most square by solution of specific analysis expression.

2. Matrix-Vector Equation(1.1)Least Squares solution

This section considers Matrix-Least Squares solution of vector type,Given MatrixA2.R$^{M £ n}$,B2.R$^{H £ K}$,C2.R$^{A £ B}$,AndD 2R$^{L £ d}$,Finding VectorX$^{The\ following}$2.R^{P}Meet

Known by the semi-Tensor Product,In Equation(1.1)Middle MatrixAWithBNumber of rows,MatrixXWithDThere is a multiple relationship between the number of rows.Can be dividedM=H; P=L;M=H; P6.=L;M6.=H; P=LAndM6.=H; P6.=LFour situations to consider the problem.VerifiedM=H; P=LAndM=H; P 6=LUnder the circumstances$X^{The following}$Same parse expression,LikewiseM 6=H; P=LAndM 6=H; P 6=LSimilar results in scenario,So just fromM=HAndM 6=HIn both cases, discuss.Our research thought is,First of all, the problem is transformed into the least square problem under the ordinary product by the definition of semi-tensor product.,Combined with the differential and generalized inverse of matrix, the least square solution is given.$X^{The following}$Specific parse expression.

2.1 Simple EormM=H

Defined by semi-Tensor Product,Available questions(2.1)InM=HNecessary Condition for solution in case.Lemma1.IfXIs the problem(2.1)The solution,Canonical matrixA. to B. to C. to D.Dimension needs to be satisfiedI)$^N{}_K$And$_A{}^L$Must be a positive integer.

PT_1=L;$^{-BT}{}_A{}^1$=D

To$_A{}^L$Will be positive integer.In additionP=$_A{}^L$=$^N{}_K$B=D.In well syndrome.

Said Lemma1In the conditions for dimension compatible conditionsAnd assume that problem(2.1)Meet compatible conditions.Will(2.1)-Transformation

WhichT_1= LCMFN; PG; t_2= LCMF1; AG.Remember(2.2)-The target functionF(X).At this time have

MinF(X)

$Min^{XK}K[A_RA_{KR}¢A_{(P ¡1)KR}]X¡B_RK^{2XAXB}KC_{IJ}X¡[D_{IJ}D_{AI; j}¢D_{(P ¡1)AI; j}]^TK^2;X2R^P$

R= 1I= 1J= 1

So only the most square by Solution$X^{¤}$= [0:0332;0:2373;0:2391]^T.

2.2 General situationM6=H

By semi-Tensor Product DefinitionAvailableM=6HWhen Problem(2.1)Of dimension compatible conditions.Lemma2IfXIs problem(2.1)OF SOLUTIONThe MatrixA; B; C; dOf dimension shall be meetI)$_M{}^{HN}{}_K$And$_A{}^L$Will be positive integerCommunist PartyFK;$_M{}^H$G= 1.

P=$_A{}^L$=$_{Mk}{}^{NH}$B=D.

SyndromeBy semi-the definition of the tensor products of have

3. Matrix-Matrix Equation(1.1)Least Squares solution

Modeled on section II,Also pointsM=H,L=P;M=H; L6.=P;M6.=H; L=PAndM6.=H; L6=PFour of the discussed problem.First considerM=HL=PThe situationOther situation can be similar discussion.By semi-the definition of the tensor products of Will(3.1)-Transformation for ordinary product under of the most square by ProblemAnd then combined with Singular Value Decomposition and generalized inverse given the most square by Solution$X^{¤}$Of analysis expression.For convenient remember(3.1)-In right minimum problem in target functionG(X):

3.1 Simple situationM=H; P=L

Lemma3IfX2$R^{P £ Q}$Is problem(3.1)OF SOLUTIONThe coefficient matrixA; B; C; dOf dimension meet the following conditions(I)$^N{}_L{}^D{}_B$Will be positive integer.MinG(X)

By the first-order differential Necessary Conditions,FunctionG($X^{The following}$)Minimum Required$^{@ G(X)}$= 0:Therefore

Among them$U_{1.}$= [$U_{11}U_{12}$],$V_{1.}$= [$V_{11}V_{12}$]; $U_{2.}$= [$U_{21.}U_{22.}$]$V_{2.}$= [$V_{21.}V_{22.}$]All-column orthogonal matrix.But§$_{1.}$=

Diag (1.1.1.2.$_{Ã ¡^{\circ} ¡}$My wife and I$_{Ã ¡^{\circ} ¡}$1.S),§2.= Diag ('1.$_{Ã ¡^{\circ} ¡}$'2.My wife and I'T),Among them$_{Ã ¡^{\circ} ¡}$1.I0;'J0 (I= 1;My wife and I; S;J= 1;My wife and IT)Matrix respectivelyA; cSingular Value, Rank (A) =S, Rank (C) =T:

Theorem3.Given MatrixA2.$R^{M £ n}$,B2.$R^{H £ K}$,C2.$R^{A £ B}$AndD2.$R^{L £ d}$,Canonical Equation(3.1)InM=H; P=LThe least squares solution in the case$X^{The following}$The parse expression is

3.2 General situationM=H; P=6L

By semi-Tensor Product DefinitionM=H; P=6 LSituation under Problem(3.1)Of dimension compatible conditions.

Lemma4IfX2R$^{P \pounds Q}$Is problem(3.1)Of the most square by SolutionThe shall be meet two conditions

(I)$^{D}{}_{B}$Will be positive integer;

P=-^{L}=$_{®}$^{N}Q=$^{Ad}{}_{B}$ -=$_{®}$^{K}Which®And¯=61RespectivelyNAndKAndLAndAOf Common DivisorAndCommunist PartyF ¯;$^{D}{}_{B}$G= 1.

SyndromeBy semi-the definition of the tensor products of have

From Lemma4Can be seen in its most square by solution not onlyIts all solution called compatible solution.Assume that problem4Meet Lemma4The number of dimensions in the compatible conditionsThe available the following results:ForANX=BIts results similar3.1Section

Repeat3:1Section derivation of process available the following theorem.

3.3 General situationM=6H; P=L

Same first givenM 6=H; P=LSituation under Problem(3.1)Of dimension compatible conditions.Lemma5IfX2R$^{P \pounds Q}$Is problem(3.1)Of the most square by SolutionThe shall be meetI)$_{M}$^{H}And$^{D}{}_{B}$Must be positive integer;

Ii)P=L=$_{®}$$^{N}{}_{M}{}^{H}$Q=$^{Ad}{}_{B}$=$_{®}$^{K};Which®IsNAndKOf Common DivisorAndCommunist PartyF®;$_{M}$^{H}G= 1:

SyndromeProve process similar to Lemma4.

¡

Similar3.1Section.RememberA= (A-I$_{H=m}$),C= (C-I$_{D=B}$),The and transformationM=H; P=LThe situation.

WhichP=L=$_{M®}$^{NH}.Repeat3.1Section of ProcessThe have the following results.

Theorem5A given matrixA2R$^{M \pounds n}$B2R$^{H \pounds K}$C2R$^{A \pounds B}$AndD2R$^{L \pounds d}$.AAndBThe Singular Value Decomposition such(3.6)Shown.Then question(3.1)InM6.=HThe least squares solution in the caseX$^{\text{The following}}$Can be expressed

X$^{\text{The following}}$

3.4 General situationM6.=H,P6.=L

M=6 h,P=6 LSituation Problem(3.1)The dimension compatibility condition.Lemma6.If Problem(3.1)Have least squares solutionX2.R$^{P\pounds q}$,Meet − −

I)$_{M}$^{H}And$^{D}{}_{B}$Positive Integer.P=^{L}=$_{®}$^{N}Wang Yi$_{M}$^{H},Q=$_{®}$^{K}=-^{A}¢$^{D}{}_{B}$Which®And¯=61RespectivelyNAndKAndA

LOf Common Divisor;

Communist PartyF®;$_{M}$^{H}G= 1;Communist PartyF ¯;$^{D}{}_{B}$G= 1.

SyndromeBy semi-tensor product of definition

SoCommunist PartyF®;$_{M}$^{H}G= 1AndCommunist PartyF ¯;$^{D}{}_{B}$G= 1.Syndrome.

Assume that(3.1)Meet Lemma6The number of dimensions in the compatible conditions.RememberA= (A-I$_{H=m}$).At this timeTransformationM=H; P6=LOf FormRepeat3.2Section of ProcessThe has the following results.

Theorem6A given matrixA2R$^{M \pounds n}$B2R$^{H \pounds K}$C2R$^{A \pounds B}$AndD2R$^{L \pounds d}$AAndBThe Singular Value Decomposition such(3.6)Shown in.The Problem(3.1)InM6=H; P6=LSituation under of the most square by SolutionX$^{¤}$Can be said

4. Conclusion

Semi-tensor product as an a kind of new of research tools in more linear function, power system, Boolean Network, cryptography and fuzzy control and other fields have a wide range of application.In this paper, the semi-tensor product matrix equation solving problemStudy the semi-tensor product under matrix equationsAX=B; XC=DOf the most square by SolutionPoints Matrix-Vector Equation(X 2R^{P})And Matrix-Matrix Equation(X2R$^{P \pounds Q}$)Two kind of situation the discussion.Combined with semi-tensor product of definition for the solvability of the problem compatible conditionsThe coefficient matrix dimension to meet of relationshipAnd then will be semi-tensor product under the problem transformation for ordinary matrix product under the matrix equation the most square by ProblemCombined with Generalized InverseMatrix Differential and Singular Value Decomposition give the specific analytical expression of the solution of the problem.A simple numerical example is given for each case to verify the correctness of the theoretical results.

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