Dynamic Analysis of WeChat Rumor Propagation

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Abstract: Considering that WeChat users are divided into three categories: not reading rumors, reading rumors but not propagating, and reading and spreading, a dynamic model of WeChat rumor propagation is established, and the existence conditions of the model equilibrium point are analyzed, which proves the global stability of the rumor-free propagation point and gives the key factors of control.

Keywords: WeChat Rumors; Infectious Disease Models; Stability Analysis; Balance Point

1. Introduction

According to the 51st China Internet Development Statistics Report, as of December 2022, the size of China's internet users reached 1.51billion, and the internet penetration rate reached 74.4%[1]. With the rapid development of internet technology, the way of rumor dissemination has gradually evolved from word of mouth between people to new media social platforms such as Weibo, WeChat and Tik Tok, and has the characteristics of fast spread, wide impact and high degree of harm, so it is of great significance to discuss how to curb the spread of WeChat rumor, this paper will use the modeling idea of the famous Kermack-Mckendrick SIR "barn" model in infectious disease dynamics[2]. The dynamic model of WeChat rumor propagation was established, and the propagation mechanism of WeChat rumor was analyzed.

2. Model formulation

Consider, divide WeChat users into three categories, $S(t)$ means that WeChat users who have not read false news and are likely to spread once they read it are called susceptible people in infectious diseases, $I(t)$ means WeChat users who forward and spread false news (such as infected people in the infection) are called the transmitted population, and $R(t)$ means WeChat users who read but do not spread (such as recoverers in infectious diseases) are called rational people.k is the maximum capacity of the environment, that is, the density of WeChat users in a specific environment, α is the internal growth rate, that is, the maximum increase rate of WeChat users in a specific environment, β is the contact rate, that is, the rate of contact with false news and spread, ε is the rate at which the forwarded false message no longer has an impact, f is the rate of users who exit WeChat.

Then the propagation model of WeChat rumor is as follows:

$$\begin{cases} \dfrac{dS}{dt} = rS(1-\dfrac{S}{k}) - \dfrac{\beta SI}{1+\alpha S} \\ \dfrac{dI}{dt} = \dfrac{\beta SI}{1+\alpha S} - eI - \varepsilon I \\ \dfrac{dR}{dt} = eI - \varepsilon R \end{cases} \qquad (1)$$

Given the practical significance, we only consider on

$$R^3{}_+ = \{(X, S, I) \in R^3 : X \ge 0, S \ge 0, I \ge 0\}$$

Lemma 1 Ω is the global attractor of the system (1) in R_+^3 and the forward invariant set of the system.

System (1) has non-negative equilibrium points:

$E_0=(0,0,0)$ $E_1=(k,0,0)$, $E_2=\left(S^*,\frac{r(1-\frac{S^*}{k})(1+\alpha S^*)}{\beta},\frac{eI^*}{\varepsilon}\right)$, $S^*=\frac{e+\varepsilon}{\beta-\alpha(e+\varepsilon)}$ Equilibrium points E_0, E_1 b

always exist; If $\frac{\beta}{\alpha(e+\varepsilon)}<1$ A>1 equilibrium point E_2 is present .

3. Stability analysis

Theorem 1 Take $R_0=\frac{\beta k}{(1+\alpha k)(e+\varepsilon)}$, when $R_0<1$, E_1 is locally progressively stable, and when $R_0>1$, E_1 is unstable.

Prove The Jacobi matrix of system (1) at E_1 is

$$\begin{pmatrix} -r & 0 & 0 \\ 0 & \frac{\beta k}{1+\alpha k}-e-\varepsilon & 0 \\ 0 & e & -\varepsilon \end{pmatrix},$$

Then the characteristic equation corresponding to $J(E_1)$ is

Then the corresponding characteristic equation is:

$$(\lambda+r)\left(\lambda-(\frac{\beta k}{1+\alpha k}-e-\varepsilon)\right)(\lambda+\varepsilon)=0.$$

Solved $\lambda_1=-r<0$, $\lambda_2=\frac{\beta k}{1+\alpha k}-e-\varepsilon$ $\lambda_3=-\varepsilon<0$ .When $\frac{\beta k}{(1+\alpha k)(e+\varepsilon)}<1$, λ_1 λ_2 and λ_3 all realnumbers;

When $\frac{\beta k}{(1+\alpha k)(e+\varepsilon)}>1$ the characteristic equation has a positive real root. So take $R_0=\frac{\beta k}{(1+\alpha k)(e+\varepsilon)}$ .When $R_0<1$, E_1 is locally progressively stable; When $R_0>1$, E_1 is unstable.

Theorem 2 For system (1), when $r>\frac{2r}{k}+\frac{\beta I^*}{S^*(1+\alpha S^*)^2}$, the equilibrium point E_2 is unstable; When $r<\frac{2r}{k}+\frac{\beta I^*}{S^*(1+\alpha S^*)^2}$,the equilibrium point E_2 is locally progressive and stable.

Prove The E_2 Jacobi matrix at the system (1) is

$$\begin{pmatrix} rS-\frac{2rS}{k}-\frac{\beta I}{(1+\alpha S)^2} & \frac{\beta S}{1+\alpha S} & 0 \\ \frac{\beta I}{(1+\alpha S)^2} & -(e+\varepsilon) & 0 \\ 0 & e & -\varepsilon \end{pmatrix},$$

Then the characteristic equation corresponding to $J(E_2)$ is:

$$\left[\lambda-\left(rS-\frac{2rS}{k}-\frac{\beta I}{(1+\alpha S)^2}\right)\right](\lambda+\varepsilon)[\lambda+(e+\varepsilon+\frac{\beta^2 SI}{(1+\alpha S)^3})]=0.$$

Because $\lambda_1=rS-\frac{2rS}{k}-\frac{\beta I}{(1+\alpha S)^2}<0$, when $r<\frac{2r}{k}+\frac{\beta I^*}{S^*(1+\alpha S^*)^2}, \lambda_2=-\varepsilon$

So when $r>\frac{2r}{k}+\frac{\beta I^*}{S^*(1+\alpha S^*)^2}$ the equilibrium point E_2 isunstable;when $r<\frac{2r}{k}+\frac{\beta I^*}{S^*(1+\alpha S^*)^2}$, the equilibrium point E_2 is locally progressive stable.

Theorem 3 When, the equilibrium point $E_1\left(k,0,0\right)$ is global progressive and stable

From theorem 1, it is known that when the equilibrium point E_1 of $R_0<1$ is locally asymptotically stable, then the 2 equation of system (1) is: $\frac{dI}{dt}=I(\frac{\beta SI}{1+\alpha S}-e-\varepsilon)<0$ So there is $\lim_{t\to 0} I(t)=0$. It can be seen from the limit system theory that systems (1) and $\begin{cases} \frac{dS}{dt}=rS(1-S)=M(S,R) \\ \frac{dR}{dt}=eI-\varepsilon R=N(S,R) \end{cases}$ (2)

have equivalent kinetic properties , and taking the Dulac function $B=\frac{1}{SR}$ then there is $\frac{\partial(MB)}{\partial S}+\frac{\partial(NB)}{\partial R}<0$ so system (2) is in $\Omega_1=\{(S,I):S>0.I>0\}$ There is no closed line inside. That is, $E_1^{'}(k,0)$ is globally progressive stable in Ω_1. So when $R_0<1$, the equilibrium point $E_1(k,0,0)$ of system (1) is global progressive and stable.

4. Conclusion

This paper mainly discusses the dynamic model of false news propagation in WeChat, divides WeChat users into three categories, discusses its propagation mechanism by using the differential equation model, obtains the existence conditions of the positive equilibrium point, analyzes the local stability conditions of the positive equilibrium point, and proves that there is no false news propagation point E_1 Global stability.Thresholds that determine whether false information spreads in susceptible people (similar to the basic regeneration number in an infectious disease model) are obtained R_0 , take $R_0 = \frac{\beta k}{(1+\alpha k)(e+\varepsilon)}$, if $R_0 < 1$, the number of rumor propagation tends to zero and no longer spreads

From the threshold $R_0 = \frac{\beta k}{(1+\alpha k)(e+\varepsilon)}$, it can be seen that if the system capacity k and the internal growth rate α are fixed, the result of rumor propagation is ultimately determined by the removal rate e , ε and contact rate β of WeChat users. If the system capacity k , internal growth rate α , and contact rate β are fixed, the more emigration rates e and ε (rational WeChat users), the less the number of rumors will spread, and eventually it will tend to zero; If the system capacity k , the internal growth rate α , the emigration rate e and ε are fixed, the smaller the contact rate β , the less the number of rumors will spread, and eventually it will tend to zero.

Therefore, in order to curb the spread of false news, it is necessary to rationally increase WeChat users and reduce the contact rate of spreading false news among WeChat users. Generally, official public accounts can be used to publicize and refute rumors, especially for some emergencies, it is necessary to publish the facts in time and strangle false news in the cradle.

References

[1] China Internet Network Information Center. The 50th Statistics on Internet Development in China report [R].[2022-09-01]

[2] Ma ZE, Zhou YC, etc. Mathematical modeling and research of infectious disease dynamics[M]. Scientific publishing, 2015, 7.

[3] Huang YX, Wang H. Stability studies of eco-epidemiological diseases with bait-eating diseases[J]. Chinese Journal of Biomathematics, 2008, 23(1): 132-138.

[4] Yang R, Gao JG. Model analysis of a class of ecological epidemiology with saturation infection rate [J]. Journal of Beihua University: Natural Science Edition, 2012, 13(4): 387-390.

[5] Huo LA, Ma CY. Optimal control of rumor spreading model with consideration of psychological factors and time delay[J]. Discrete Dyn Nat Soc, 2018, 2018: 9314907.

[6] Sun LL. Research on Network Rumor Propagation and Control Based on Infectious Disease Model [J]. Journal of Nanjing Normal University, 2017,6: 70-77.

[7] Gao WG, Cai YL. Dynamic modeling and equilibrium stability analysis of networks rumor propagation [J].Journal of Yangzhou University (Natural Science Edition) 2022, 5(25) :7-12.

[8] Zhu LH, Yang F, Guan G, et al. Modeling the dynamics of rumor diffusion over complex networks [J]. Inform Sciences, 2021, 562: 240-258.

Acknowledgement

This work was supported by Sichuan Minzu College(XYZB2106ZB)